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SOME GENERAL RESULTS IN ELEMENTARY NUMBER THEORY

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Abstract. Some results in Elementary Number Theory are generalized. All the proofs use the principle of mathematical induction. Induction on two or even three distinct variables are involved in each case. In order to save space the details of the proofs are omitted in many cases.

Key words:- Mathematical Induction, Infinity.
AMS Subject Classification No:- 11A99.

1. Some simple results

Theorem 1.1.

1) $0+1+2+3+\cdots+(1n-1)^2 = n^2 - 1\left(\frac{n(n+1)}{2}\right)$
(Standard result 1,2)

2) $1+3+5+7+\cdots+(2n-1)^2 = n^2 + 0\left(\frac{n(n+1)}{2}\right)$
(Standard result 1,2)

Trivially generalizing we can prove by Mathematical Induction

3) $2+5+7+9+\cdots+(3n-1)^2 = n^2 + 1\left(\frac{n(n+1)}{2}\right)$

4) $3+7+11+15+\cdots+(4n-1)^2 = n^2 + 2\left(\frac{n(n+1)}{2}\right)$
....

ξ) $(\xi-1)+(2\xi-1)+\cdots+(\xi n-1)^2 = n^2 + (\xi-2)\left(\frac{n(n+1)}{2}\right)$ (ξ +ve integer)

$$\sum_{j=1}^{n}(\xi j-1)^2 = n^2 + (\xi-2)\left(\frac{n(n+1)}{2}\right) \quad (\xi = 1,2,3,\cdots\infty).$$

Re-writing in the notation for the super-sums (Sum level indicated on top) we have

$$\left[(\xi \overset{[1]}{n} - 1)^2\right] = (\xi n-1)^2$$

$$S_n\left[(\xi \overset{[1]}{n} - 1)^2\right] = n^2 + (\xi-2)\left(\frac{n(n+1)}{2}\right) \tag{1}$$

$$= \left[(\xi \overset{[2]}{n} - 1)^2\right]$$

The sum of the above sequence may be curiously written as

$$
\begin{aligned}
S_n\left[(\xi \overset{[2]}{n} - 1)^2\right] &= (\xi - 1) + (3\xi - 2)(n-1) \\
&\quad + \left((3\xi - 1) + \xi(n - \xi)\right)(n-2) \\
&= \left[(\xi \overset{[3]}{n} - 1)^2\right] \\
S_n\left[(\xi \overset{[3]}{n} - 1)^2\right] &= (\xi - 1)n + (3\xi - 2)\left(\frac{(n-1)(n)}{2!}\right) \\
&\quad + \left((3\xi - 1) + \frac{\xi(n-\xi)(n-\xi+1)}{2!}\right) \\
&\quad \left(\frac{(n-2)(n-1)}{2!}\right) \\
&= \left[(\xi \overset{[4]}{n} - 1)^2\right] \qquad (2) \\
S_n\left[(\xi \overset{[4]}{n} - 1)^2\right] &= (\xi - 1)\left(\frac{n(n+1)}{2!}\right) \\
&\quad + (3\xi - 2)\left(\frac{(n-1)n(n+1)}{3!}\right) + \Bigg((3\xi - 1) \\
&\quad + \frac{\xi(n-\xi)(n-\xi+1)(n-\xi+2)}{3!}\Bigg) \\
&\quad \left(\frac{(n-2)(n-1)n}{3!}\right) \\
&= \left[(\xi \overset{[5]}{n} - 1)^2\right].
\end{aligned}
$$

....

$$S_n\left[(\xi n \overset{[\alpha]}{-} 1)^2\right] = (\xi - 1)\left(\frac{n(n+1)\cdots(n+\alpha-3)}{(\alpha-2)!}\right)$$
$$+ (3\xi - 2)\left(\frac{(n-1)n\cdots(n+\alpha-3)}{(\alpha-1)!}\right)$$
$$+ \left((3\xi-1) + \frac{\xi(n-\xi)(n-\xi+1)\cdots(n-\xi+\alpha-2)}{(\alpha-1)!}\right) \tag{3}$$
$$\left(\frac{(n-2)(n-1)n\cdots(n+\alpha-4)}{(\alpha-1)!}\right) = \left[(\xi n \overset{[\alpha+1]}{-} 1)^2\right].$$

(ξ +ve integer: $(n-i)$ is operative only for $(n-i) \geq 0$)

Proof. Apply the principle of mathematical induction on the variables "n", "ξ" and finally "α". □

Theorem 1.2.

$$\left[(\xi n - \overset{[1]}{(\xi - 1)})^2\right] = 1 + [\xi^2 + 2\xi](n-1) + [2\xi^2]\frac{(n-2)(n-1)}{2!}$$

$$S_n\left[(\xi n - \overset{[1]}{(\xi - 1)})^2\right] = n + [\xi^2 + 2\xi]\frac{(n-1)(n)}{2!}$$
$$+ [2\xi^2]\frac{(n-2)(n-1)(n)}{3!}$$
$$= \left[(\xi n - \overset{[2]}{(\xi - 1)})^2\right]$$

$$
\begin{aligned}
S_n\left[(\xi n - \overset{[2]}{(\xi-1)})^2\right] &= \frac{n(n+1)}{2!} + [\xi^2+2\xi]\frac{(n-1)(n)(n+1)}{3!} \\
&\quad + [2\xi^2]\frac{(n-2)(n-1)(n)(n+1)}{4!} \\
&= \left[(\xi n - \overset{[3]}{(\xi-1)})^2\right]
\end{aligned}
$$

$$
\begin{aligned}
S_n\left[(\xi n - \overset{[3]}{(\xi-1)})^2\right] &= \frac{n(n+1)(n+2)}{3!} \\
&\quad + [\xi^2+2\xi]\frac{(n-1)(n)(n+1)(n+2)}{4!} \\
&\quad + [2\xi^2]\frac{(n-2)(n-1)(n)(n+1)(n+2)}{5!} \\
&= \left[(\xi n - \overset{[4]}{(\xi-1)})^2\right]
\end{aligned}
$$

....

$$
\begin{aligned}
S_n\left[(\xi n - \overset{[\alpha]}{(\xi-1)})^2\right] &= \frac{n(n+1)\cdots(n+\alpha-1)}{\alpha!} \\
&\quad + [\xi^2+2\xi]\frac{(n-1)(n)\cdots(n+\alpha-1)}{(\alpha+1)!} \\
&\quad + [2\xi^2]\frac{(n-2)(n-1)\cdots(n+\alpha-1)}{(\alpha+2)!} \\
&= \left[(\xi n - \overset{[\alpha+1]}{(\xi-1)})^2\right]
\end{aligned}
$$

(ξ +ve integer: $(n-i)$ is operative only for $(n-i) \geq 0$).

Proof. Apply the principle of mathematical induction on the variables "n", "ξ" and finally "α". $\square$

Theorem 1.3.

$$\left[(\xi n \overset{[1]}{-} \Delta)^2\right] = [\xi - \Delta]^2 + [3\xi^2 - 2\xi\Delta](n-1) + [2\xi^2]\frac{(n-2)(n-1)}{2!}$$

$$S_n\left[(\xi n \overset{[1]}{-} \Delta)^2\right] = [\xi - \Delta]^2(n) + [3\xi^2 - 2\xi\Delta]\frac{(n-1)(n)}{2!} + [2\xi^2]\frac{(n-2)(n-1)(n)}{3!} = \left[(\xi n \overset{[2]}{-} \Delta)^2\right]$$

$$S_n\left[(\xi n \overset{[2]}{-} \Delta)^2\right] = [\xi - \Delta]^2\frac{n(n+1)}{2!} + [3\xi^2 - 2\xi\Delta]\frac{(n-1)(n)(n+1)}{3!} + [2\xi^2]\frac{(n-2)(n-1)(n)(n+1)}{4!} = \left[(\xi n \overset{[3]}{-} \Delta)^2\right]$$

$$S_n\left[(\xi n \overset{[3]}{-} \Delta)^2\right] = [\xi-\Delta]^2\frac{n(n+1)(n+2)}{3!}$$
$$+ [3\xi^2 - 2\xi\Delta]\frac{(n-1)(n)(n+1)(n+2)}{4!}$$
$$+ [2\xi^2]\frac{(n-2)(n-1)(n)(n+1)(n+2)}{5!}$$
$$= \left[(\xi n \overset{[4]}{-} \Delta)^2\right]$$

....

$$S_n\left[(\xi n \overset{[\alpha]}{-} \Delta)^2\right] = [\xi-\Delta]^2\frac{n(n+1)\cdots(n+\alpha-1)}{\alpha!}$$
$$+ [3\xi^2 - 2\xi\Delta]\frac{(n-1)(n)\cdots(n+\alpha-1)}{(\alpha+1)!}$$
$$+ [2\xi^2]\frac{(n-2)(n-1)\cdots(n+\alpha-1)}{(\alpha+2)!}$$
$$= \left[(\xi n \overset{[\alpha+1]}{-} \Delta)^2\right]$$

("ξ" +ve integer: "Δ" any integer: $(n-i)$ is operative only for $(n-i) \geq 0$).

Proof. Apply the principle of mathematical induction on the variables "n", "ξ" and finally "α". □

2. **Some simple generalizations**

Theorem 2.1. For n any +ve even integer we have

$$1.\ (n) + 2.\ (n-1) + 3.\ (n-2) + \cdots + \left(\frac{n}{2}\right)\left(\frac{n}{2}+1\right) = \frac{n(n+1)(n+2)}{12} = \frac{1}{2}\left[\sum_{\lambda=1}^{n}\sum \lambda\right].$$

Proof. Apply the principle of mathematical induction on the variable "n". □

Theorem 2.2. For n any +ve even integer and ∂ any factor of n we have

$$(\partial \cdot n) + 2\partial \cdot (n-\partial) + 3\partial \cdot (n-2\partial) + \cdots + \frac{n}{2}\left(\frac{n}{2}+\partial\right) = \frac{n^3 + 3n^2\partial + 2n\partial^2}{12\partial}.$$

Proof. Apply the principle of mathematical induction on the variable "n". □

Theorem 2.3. For all integers $n \geq 2\lambda$, $n! > n^{\lambda}$ $(\lambda = 1, 2, 3, \cdots \infty)$.

Proof. Apply the principle of mathematical induction on the variables "n" and "λ". (For $\lambda = 2$ and $\lambda = 3$ we have the standard results.) [**1**, p.9]. □

Theorem 2.4. For all $\lambda \geq 2$ and $n > 1$ ("λ" and "n" integers)

$$\frac{1}{1^{\lambda}} + \frac{1}{2^{\lambda}} + \frac{1}{3^{\lambda}} + \cdots + \frac{1}{n^{\lambda}} < 2 - \left(\frac{1}{n} + \frac{1}{n^2} + \cdots + \frac{1}{n^{(\lambda-1)}}\right).$$

Proof. Apply the principle of mathematical induction on the variables "n" and "λ". (For $\lambda = 2$ we have the standard results.) [**1**, p.10]. □

Theorem 2.5.

$$a^2 + (a+1)^2 + (a+1)^2 + (a+2)^2 + (a+2)^2 + (a+2)^2 + \cdots$$
$$(a+(n-1))^2 + (a+(n-1))^2 \cdots n \text{ times}$$
$$= a^2\left\{\frac{(n)(n+1)}{2}\right\} + 2a\left\{\frac{(n-1)(n)(n+1)}{3}\right\} +$$
$$\left\{\frac{(n-1)(n)}{2}\right\}^2 + \left\{\frac{(n-1)(n)(2n-1)}{6}\right\}$$

for all algebraic values of "a" and $n \geq 1$, n integer.

Proof. Apply the principle of mathematical induction on the variable "n". □

Theorem 2.6. For $n > 3$ integers n, $(n + 2^\lambda)$, $(n + 2^{\lambda+1})$ $(\lambda = 1, 2, 3, \cdots \infty)$ cannot all be primes.

Proof. Apply the principle of mathematical induction on the variables "n" and "λ". (For $\lambda = 1$ we have the standard result.) [**1**, p.75]. □

Theorem 2.7. For $n > 3$ integers

$$\left(n + \sum_{i=1}^{t} 2^{(\lambda+i)}\right), \left(n + \sum_{i=0}^{t} 2^{(\lambda+i)}\right), (n + 2^{(\lambda+t+1)}),$$

$[\lambda = 1, 2, 3, \cdots \infty]$, $[t = 1, 2, 3, \cdots \infty]$ cannot all be primes.

Proof. Apply Theorem 2.6 repeatedly. □

Theorem 2.8. For $n > 3$, let n, $(n+2)$, $(n+4), \cdots, (n+2\delta)$ $[\delta = 2, 3, \cdots \infty]$, be a Family of sequences of integers $\{S_\delta\}$.

Let $P = \{{}^{\delta}P_i\} = \{{}^{\delta}P_1, {}^{\delta}P_2, \cdots, {}^{\delta}P_t\}$ be the set of all odd prime numbers such that ${}^{\delta}P_i \leq \delta + 1$ for each i for each S_δ.

Then each of the sequences of $\{S_\delta\}$ $[\delta = 2, 3, \cdots \infty]$, contains utmost

$$(\delta + 1)\left[1 - \sum_{i=1}^{t} \frac{1}{{}^{\delta}P_i} + \sum_{\substack{i,j=1 \\ i \neq j}}^{t} \frac{1}{{}^{\delta}P_i\,{}^{\delta}P_j}\right] \quad \text{prime numbers.}$$

Proof. Apply the principle of mathematical induction on the variables "n" and "δ". □

Theorem 2.9. For $n > 3$, let n, $(n+2^{\xi}), \cdots (n+2^{\xi}\delta)$ $[\xi = 1, 2, 3, \cdots \infty]$, $[\delta = 2, 3, \cdots \infty]$, be a Families of sequences of integers $\{{}^{\xi}S_{\delta}\}$ for each ξ.

Let $P = \{{}^{\delta}P_i\} = \{{}^{\delta}P_1, {}^{\delta}P_2, \cdots, {}^{\delta}P_t\}$ be the set of all odd prime numbers such that ${}^{\delta}P_i \leq \delta + 1$ for each i for each S_δ.

Then each of the sequences of $\{{}^{\xi}S_{\delta}\}$ $[\xi = 1, 2, 3, \cdots \infty]$, $[\delta = 2, 3, \cdots \infty]$ contains utmost

$$(\delta+1)\left[1-\sum_{i=1}^{t}\frac{1}{\delta_{P_i}}+\sum_{\substack{i,j=1\\ i\neq j}}^{t}\frac{1}{\delta_{P_i}\delta_{P_j}}\right] \quad \text{prime numbers.}$$

Proof. Apply the principle of mathematical induction on the variables "n" and "δ" and ξ. □

Theorem 2.10. For every integer $\delta \geq 2$ and for each integer $n \geq 1$

$$(\delta!)\left|\frac{(\delta n)!}{(n!)^{\delta}}\right..$$

Proof. Apply the principle of mathematical induction on the variables "n" and "δ". (For $\delta = 2$ we have the standard result.) [**1**, p.151]. □

Theorem 2.11. For every integer $\delta_i \geq 2$ $[i = 1, 2, \cdots \tau]$ and for each integer $n \geq 1$

$$\left\{\prod_{i=1}^{\tau}(\delta_i!)\right\}\left|\frac{\prod_{i=1}^{\tau}((\delta_i n)!)}{(n!)^{\sum_{i=1}^{\tau}\delta_i}}\right..$$

Proof. Apply Theorem 2.10 on each δ_i and multiply. □

Theorem 2.12. For every integer $\delta \geq 2$ and for each integer $n_j \geq 1$ $[j = 1, 2, \cdots \epsilon]$

$$\{(\delta!)^{\epsilon}\}\left|\frac{\prod_{j=1}^{\epsilon}(\delta n_j)!}{\prod_{j=1}^{\epsilon}(n_j!)^{\delta}}\right..$$

Proof. Apply Theorem 2.10 on each n_j and multiply. □

Theorem 2.13. For every integer $\delta_i \geq 2$ $[i = 1, 2, \cdots \tau]$ and for each integer $n_j \geq 1$ $[j = 1, 2, \cdots \epsilon]$

$$\left\{\prod_{i=1}^{\tau}(\delta_i!)^{\epsilon}\right\} \left| \frac{\prod_{j=1}^{\epsilon}\prod_{i=1}^{\tau}\{(\delta_j n_j)!\}}{\prod_{j=1}^{\epsilon}\left\{(n_j!)^{\sum_{i=1}^{\tau}\delta_i}\right\}}\right. .$$

Proof. Apply Theorem 2.10 on each δ_i and n_j and multiply. □

Theorem 2.14. For every integer $\delta \geq 2$ and for each integer $n \geq 1$, if p is a prime and if $[\cdots]$ denotes the greatest integer function, the exponent of the highest power of p which divides

$$(\delta n)!/(n!)^{\delta} \text{ is } \sum_{h=1}^{\infty}\left\{[\delta n/p^h] - \delta[n/p^h]\right\}.$$

Proof. Apply the principle of mathematical induction on the variables "n" and "δ". (For $\delta = 2$ we have the standard result.) [**1**, p.151]. □

Theorem 2.15. For every integer $\delta_i \geq 2$ $[i = 1, 2, \cdots \tau]$ and for each integer $n \geq 1$, if p is a prime and if $[\cdots]$ denotes the greatest integer function, the exponent of the highest power of p which divides

$$\frac{\prod_{i=1}^{\tau}((\delta_i n)!)}{(n!)^{\sum_{i=1}^{\tau}\delta_i}} \text{ is } \sum_{i=1}^{\tau}\sum_{h=1}^{\infty}\left\{[\delta_i n/p^h] - \delta_i[n/p^h]\right\}.$$

Proof. Apply Theorem 2.14 on each δ_i and add. □

Theorem 2.16. For every integer $\delta \geq 2$ and for each integer $n_j \geq 1$ $[j = 1, 2, \cdots \tau]$ if p is a prime and if $[\cdots]$ denotes the greatest integer function, the exponent of the highest power of p which divides

$$\frac{\prod_{j=1}^{\epsilon}(\delta n_j)!}{\prod_{j=1}^{\epsilon}(n_j!)^{\delta}} \text{ is } \sum_{j=1}^{\epsilon}\sum_{h=1}^{\infty}\left\{[\delta n_j/p^h] - \delta[n_j/p^h]\right\}.$$

Proof. Apply Theorem 2.14 on each n_j and add. □

Theorem 2.17. For every integer $\delta_i \geq 2$ $[i = 1, 2, \cdots \tau]$ and for each integer $n_j \geq 1$ $[j = 1, 2, \cdots \tau]$ if p is a prime and if $[\cdots]$ denotes the greatest integer function, the exponent of the highest power of p which divides $\frac{\prod_{j=1}^{\epsilon}\prod_{i=1}^{\tau}\{(\delta_j n_j)!\}}{\prod_{j=1}^{\epsilon}\left\{(n_j!)^{\sum_{i=1}^{\tau}\delta_i}\right\}}$ is $\sum_{j=1}^{\epsilon}\sum_{i=1}^{\tau}\sum_{h=1}^{\infty}\left\{[\delta_i n_j/p^h] - \delta_i[n_j/p^h]\right\}$.

Proof. Apply Theorem 2.14 on each δ_i and n_j and add the results. □

Theorem 2.18. For every integer $\delta \geq 2$ and for each integer $n \geq 1$, in the prime factorization of $(\delta n)!/(n!)^{\delta}$, the exponent of any prime p such that $(\delta - 1)n < p < \delta n$ is equal to 1. Generalizing we may say that for every integer $\delta \geq 2$ and for each integer $n \geq 1$, in the prime factorization of $(\delta n)!/(n!)^{\delta}$, the exponent of any prime p such that $(\delta-\theta-1)n < p < (\delta-\theta)n$ is equal to $(\theta+1)$ $[\theta = 0, 1, \cdots (\delta-2)]$.

Proof. Apply the principle of mathematical induction on the variables "n" and "δ". (For $\delta = 2$ and $\theta = 0$ have the standard result.) [**1**, p.151]. □

Theorem 2.19. For every integer $\delta_i \geq 2$ $[i = 1, 2, \cdots \tau]$ and for each integer $n \geq 1$, in the prime factorization of $\frac{\prod_{i=1}^{\tau}((\delta_i n)!)}{(n!)^{\sum_{i=1}^{\tau}\delta_i}}$, the exponent of any prime p such that $\left(\left\{\prod_{i=1}^{\tau}(\delta_i)\right\} - 1\right) n < p < \left\{\prod_{i=1}^{\tau}(\delta_i)\right\} n$ is equal to 1.

Generalizing we may say that for every integer $\delta_i \geq 2$ $[i = 1, 2, \cdots \tau]$ and for each integer $n \geq 1$, in the prime factorization of $\frac{\prod_{i=1}^{\tau}((\delta_i n)!)}{(n!)^{\sum_{i=1}^{\tau}\delta_i}}$, the exponent of any prime p such that $\left(\left\{\prod_{i=1}^{\tau}(\delta_i)\right\} - \theta - 1\right) n < p < \left(\left\{\prod_{i=1}^{\tau}(\delta_i)\right\} - \theta\right) n$ is equal to $(\theta+1)$ $[\theta = 0, 1, \cdots \left(\left\{\prod_{i=1}^{\tau}(\delta_i)\right\} - 2\right)]$.

Proof. Apply Theorem 2.18 for every integer $\delta = \left\{\prod_{i=1}^{\tau}(\delta_i)\right\}$, $(\delta \geq 2)$. □

Theorem 2.20. For every integer $\delta \geq 2$ and for each integer $n_j \geq 1$ $[j = 1, 2, \cdots \tau]$, in the prime factorization of $\frac{\prod_{j=1}^{\epsilon}(\delta n_j)!}{\prod_{j=1}^{\epsilon}(n_j!)^{\delta}}$, the exponent of any prime p such that $(\delta-1)\left\{\prod_{j=1}^{\epsilon}(n_j)\right\} < p < \delta\left\{\prod_{j=1}^{\epsilon}(n_j)\right\}$ is equal to 1.

Generalizing we may say that for every integer $\delta \geq 2$ and for each integer $n_j \geq 1$ [$j = 1, 2, \cdots \tau$], in the prime factorization of $\frac{\prod_{j=1}^{\epsilon}(\delta n_j)!}{\prod_{j=1}^{\epsilon}(n_j!)^{\delta}}$, the exponent of any prime p such that $(\delta-\theta-1)\left\{\prod_{j=1}^{\epsilon}(n_j)\right\} < p < (\delta-\theta)\left\{\prod_{j=1}^{\epsilon}(n_j)\right\}$ is equal to $(\theta+1)$. [$\theta = 0, 1, \cdots (\delta-2)$].

Proof. Apply Theorem 2.18 for every integer $n = \left\{\prod_{j=1}^{\epsilon}(n_j)\right\}$ and $\delta \geq 2$. □

Theorem 2.21. For every integer $\delta_i \geq 2$ [$i = 1, 2, \cdots \tau$] and for each integer $n_j \geq 1$ [$j = 1, 2, \cdots \tau$], in the prime factorization of $\frac{\prod_{j=1}^{\epsilon}\prod_{i=1}^{\tau}\{(\delta_j n_j)!\}}{\prod_{j=1}^{\epsilon}\left\{(n_j!)^{\sum_{i=1}^{\tau}\delta_i}\right\}}$, the exponent of any prime p such that

$$\left(\left\{\prod_{i=1}^{\tau}(\delta_i)\right\}-1\right)\left\{\prod_{j=1}^{\epsilon}(n_j)\right\} < p < \left\{\prod_{i=1}^{\tau}(\delta_i)\right\}\left\{\prod_{j=1}^{\epsilon}(n_j)\right\}$$

is equal to 1.

Generalizing we may say that for every integer $\delta_i \geq 2$ [$i = 1, 2, \cdots \tau$] and for each integer $n_j \geq 1$ [$j = 1, 2, \cdots \tau$], in the prime factorization of $\dfrac{\prod_{j=1}^{\epsilon}\prod_{i=1}^{\tau}\{(\delta_j n_j)!\}}{\prod_{j=1}^{\epsilon}\left\{(n_j!)^{\sum_{i=1}^{\tau}\delta_i}\right\}}$, the exponent of any prime p such that

$$\left(\left\{\prod_{i=1}^{\tau}(\delta_i)\right\}-\theta-1\right)\left\{\prod_{j=1}^{\epsilon}(n_j)\right\}<p<\left(\left\{\prod_{i=1}^{\tau}(\delta_i)\right\}-\theta\right)\left\{\prod_{j=1}^{\epsilon}(n_j)\right\}$$

is equal to $(\theta+1)$. [$\theta = 0, 1, \cdots \left(\left\{\prod_{j=1}^{\tau}(\delta_i)\right\}-2\right)$].

Proof. Apply Theorem 2.18 for every integer $\delta = \left\{\prod_{i=1}^{\tau}(\delta_i)\right\}$, $\delta \geq 2$ and for every integer $n = \left\{\prod_{j=1}^{\epsilon}(n_j)\right\}$. □

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